

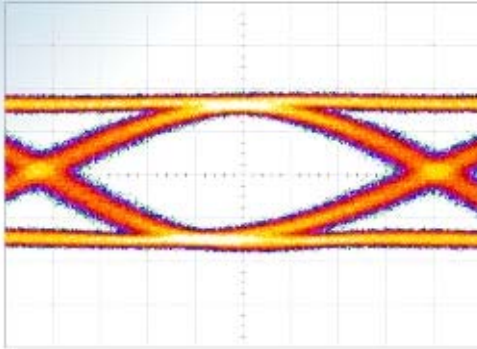


SHF Communication Technologies AG

Wilhelm-von-Siemens-Str. 23 • Aufgang D • 12277 Berlin – Marienfelde • Germany

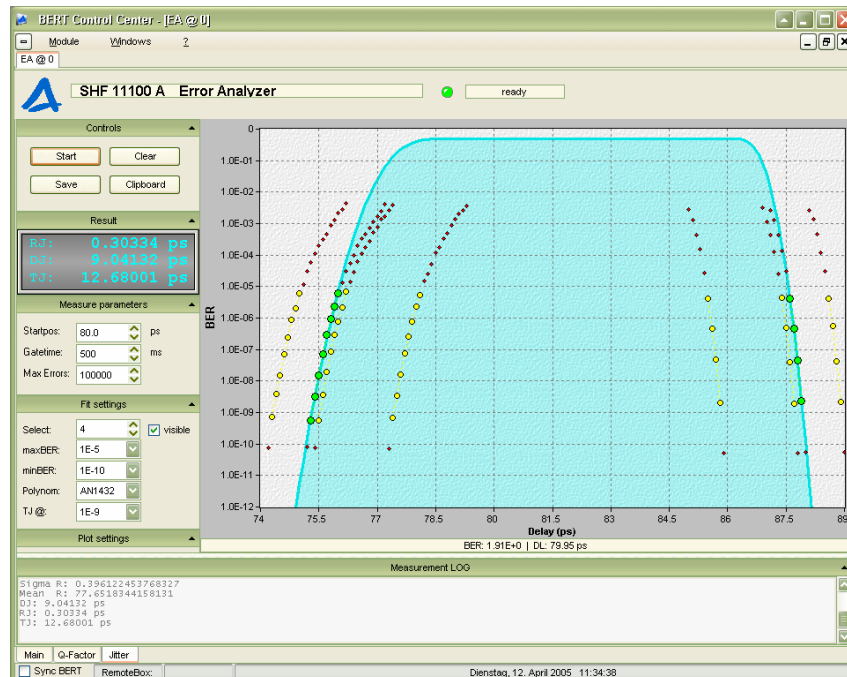
Phone ++49 30 / 772 05 10 • Fax ++49 30 / 753 10 78

E-Mail: sales@shf.biz • Web: <http://www.shf.biz>



Application Note AN-JITTER-1

Jitter Analysis using SHF 10000 Series Bit Error Rate Testers





Introduction

This application note introduces the technique and theory of using the SHF 1000 series of bit error rate testers for jitter analysis. The SHF 11100 A error analyzer has a precise delay line (100 fs step size) and the corresponding control software is able to measure the BER as a function of the time position of the decision point. Applying a fitting technique to the data allows the extraction of random and deterministic jitter components. Jitter analysis is one of the most commonly performed tests on high-speed digital communication components.

Jitter

Jitter is the short term variation of the signal from its ideal location in time. With increasing data rates, jitter related problems become more apparent. A simple classification of jitter is shown in Figure 1.

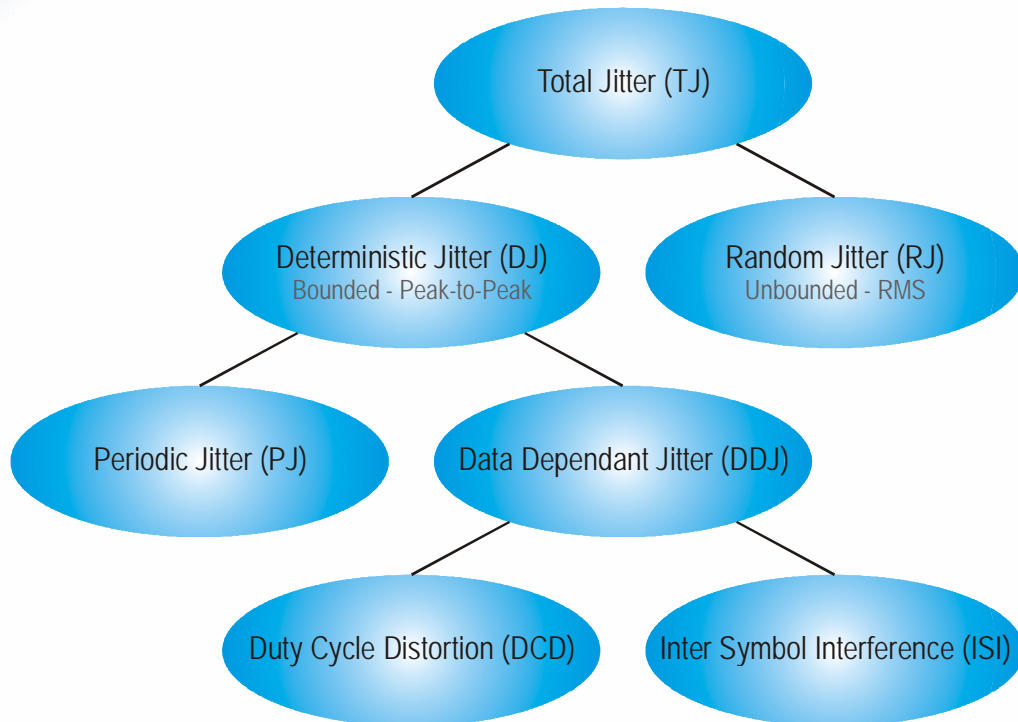


Figure 1 - Jitter Components

Total jitter (TJ) is composed of several independent parts which can be separated into deterministic (DJ) and random (RJ) components. The method described here allows only the extraction of these two components. A further separation of the deterministic parts is not possible using this method.

Random jitter has an unbounded Gaussian distribution. It is caused by noise processes (thermal, shot, flicker). Since it is Gaussian in nature, the distribution is quantified by the standard deviation (σ) and mean (μ). The infinitely extended tails of the distribution are used to fit a Gaussian function and to extract σ and μ . The peak-to-peak value of random jitter must always be specified for a certain bit error rate.

Deterministic jitter is the non-Gaussian distributed component of the total jitter of a signal. As shown in Figure 1, it is composed of periodic jitter and data dependant jitter. It is characterized by its peak-to-peak value.

A more detailed treatment of jitter theory and fundamentals can be found in the literature.



BER Scan Theory

Figure 2 shows a typical eye diagram. The ideal decision point is in the center of the eye and is marked by the red dot. In general, the lowest bit error rate is to be expected at this point. If the decision point is moved towards the right or left crossing point, the probability of an error increases steadily.

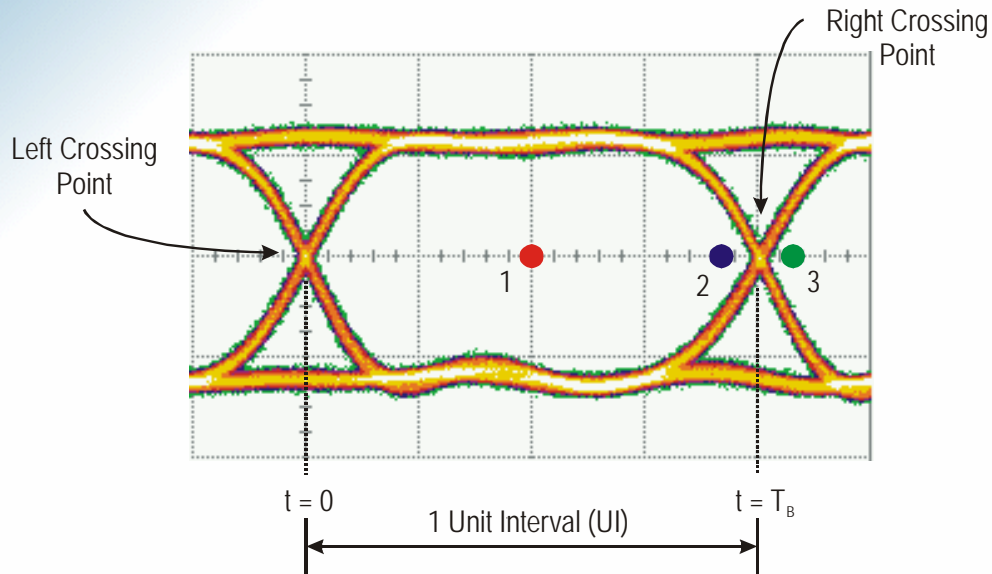


Figure 2 - Eye Diagram

To determine the jitter components, the time position of the decision point is moved from one side of a crossing point (blue dot, point 2) to the other side (green dot, point 3). At the same time, the BER is measured for each step. As the decision point is stepped from the center of the eye towards the crossing point, the BER increases continuously. At some point, the error analyzer will lose synchronization. The theoretical BER maximum is 0.5 for a typical pseudo-random bit sequence with an equal number of "1"s and "0"s (equal transition density).

Figure 3 shows a typical measurement. This measurement was performed using an SHF 11100 A error analyzer; the decision point was stepped in 100 fs steps along the crossing point of a 50 Gbps data signal from an SHF 12100 A bit pattern generator. Both instruments were connected back-to-back with a 0.5 m long 50 GHz microwave cable assembly.

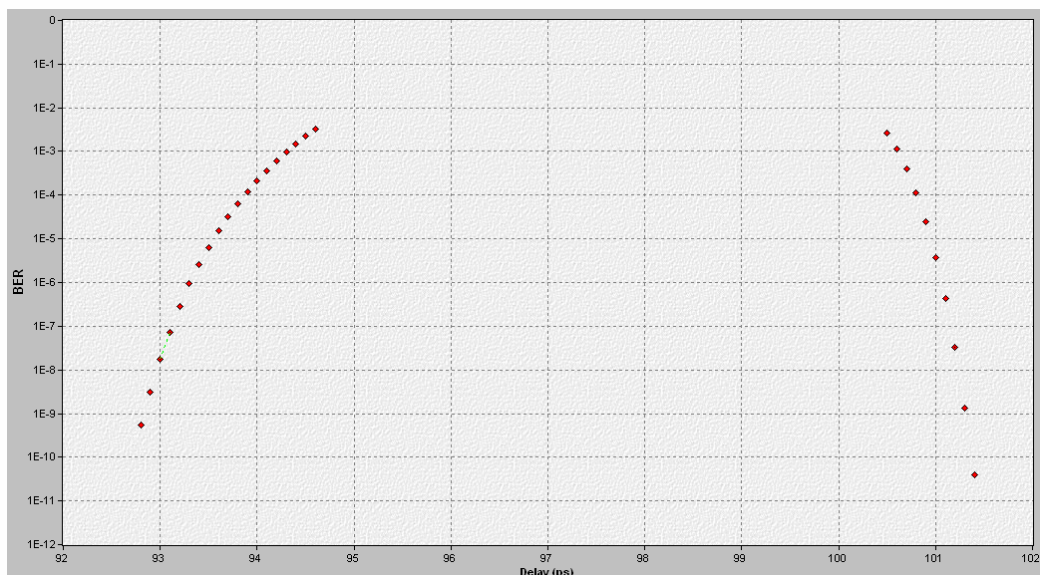


Figure 3 - Typical BER Scan

To extract the deterministic and random jitter components from the measurements, the "Dual Dirac Model" or "Double Delta Model" is employed [1,3]. In this model, the probability density function



(PDF) for the deterministic jitter component is represented by two Dirac functions separated in time by the peak to peak value DJ = $\mu_L - \mu_R$.

$$PDF_{DJ} = \frac{1}{2} [\delta(t - \mu_R) + \delta(t - \mu_L)] \quad (1)$$

Random jitter is modeled as a Gaussian function with a standard deviation $\sigma_{R/L}$.

$$PDF_{RJ} = \frac{1}{\sqrt{2 \cdot \pi} \sigma} e^{-\frac{t^2}{2\sigma^2}} \quad (2)$$

The total jitter probability density function can be determined by convoluting the PDFs of DJ and RJ.

$$PDF_{TJ} = PDF_{DJ} \otimes PDF_{RJ} \quad (3)$$

$$= \frac{1}{2\sqrt{2 \cdot \pi}} \left(\frac{1}{\sigma_R} e^{-\frac{(t-\mu_R)^2}{2\sigma_R^2}} + \frac{1}{\sigma_L} e^{-\frac{(t-\mu_L)^2}{2\sigma_L^2}} \right)$$

Equation 3 is illustrated in Figure 4. Since this model is quite simple with respect to the complex processes and contributions, it is necessary to select the regions for the extraction of the random jitter parameters carefully. While the distribution is dominated by deterministic jitter in the vicinity of the crossing point, it becomes dominated more and more by random jitter with increasing distance. Hence it is important to collect measurement data further away from the crossing point, i.e. towards the tails of the distribution where it is expected to be purely Gaussian.

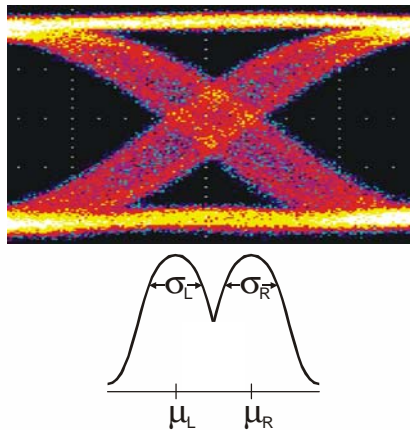


Figure 4 – Eye Crossing point with probability density function (PDF)

As mentioned above, Equation 3 represents the probability density function of the jitter model being used. The simplified mathematical relation between the probability density function and the BER is as follows:

$$BER_L(t) = \rho_L \cdot \int_{-\infty}^t PDF_{TJ}(t) dt \quad (4)$$

$$BER_R(t) = \rho_R \cdot \int_t^{\infty} PDF_{TJ}(t) dt \quad (5)$$

Equations 4 and 5 represent the cumulative distribution function (CDF) which is the time integral of the probability density function. ρ_L and ρ_R are the average transition density, that is the ratio of the number of logic transitions to the total number of transmitted bits. For pseudo-random bit sequences, this ratio is 0.5.



$$\operatorname{erfc}(t) = 1 - \operatorname{erf}(t) = \frac{2}{\sqrt{\pi}} \cdot \int_t^{\infty} e^{-x^2} dx \quad (6)$$

Following the steps given in Appendix A, we get the following expressions:

$$\begin{aligned} \operatorname{BER}_L(t) &= \rho_L \cdot \int_{t'=-\infty}^t \operatorname{PDF}_{TJ}(t') dt' \\ &= N_L \cdot \operatorname{erfc}\left(\frac{\mu_L - t}{\sqrt{2} \cdot \sigma_L}\right) \text{ with } N_L = \frac{\rho_L}{2} = 0.25 \end{aligned} \quad (7)$$

$$\begin{aligned} \operatorname{BER}_R(t) &= \rho_R \cdot \int_{t'=t}^{\infty} \operatorname{PDF}_{TJ}(t') dt' \\ &= N_R \cdot \operatorname{erfc}\left(\frac{t - \mu_R}{\sqrt{2} \cdot \sigma_R}\right) \text{ with } N_R = \frac{\rho_R}{2} = 0.25 \end{aligned} \quad (8)$$

BER Scan Measurement

The SHF 11100 A error analyzer performs all the necessary steps to extract the random and deterministic jitter components automatically.

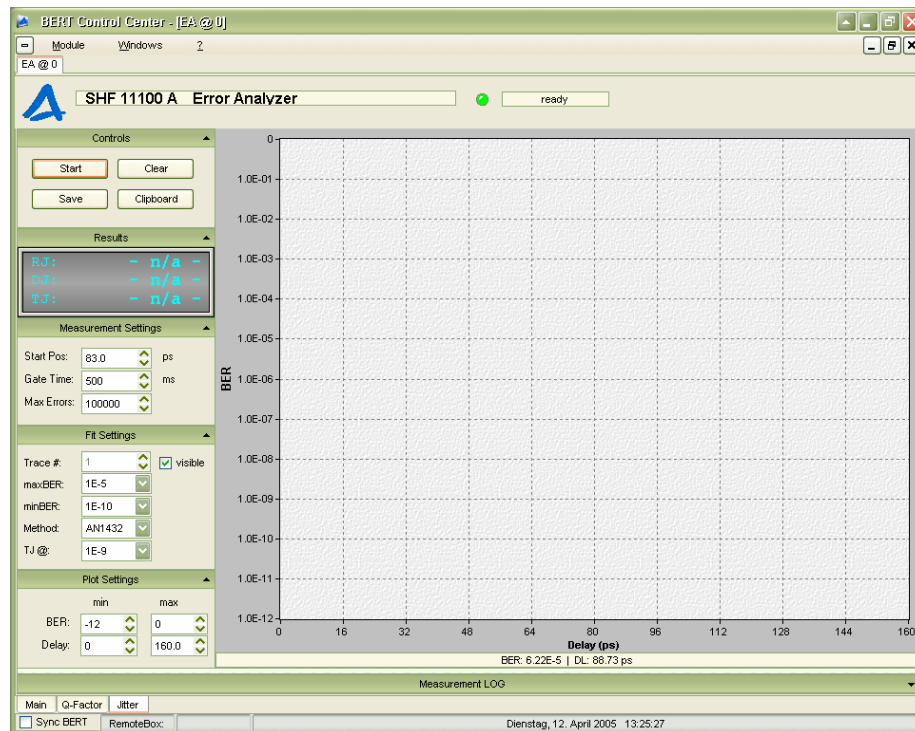


Figure 5 – Jitter Analysis Display

If the jitter analysis is selected for the first time it will start with an empty diagram (Figure 5). Several parameters corresponding to the measurement and the data fitting can be set by the user.



Measurement Settings:

Start Pos: Determines where (time delay) the algorithm starts with the measurement. It should be chosen well within the low BER range of the eye. If an *Auto Search* is performed in advance, a suitable time delay starting position close to the right crossing of the eye is automatically chosen. Alternatively, a different starting point can be entered by the user.

Gate Time: Determines the maximum gate time which is employed during the whole analysis. To measure very low bit rates, the gate time needs to be adjusted to correspondingly high values, e.g. at a bit rate of 50 Gbps, a gate time of 2000 ms should be selected to take a BER of $1 \cdot 10^{-10}$ into consideration, if a confidence of at least 10 errors is required. Note that selecting a long gate time will slow down the jitter analysis considerably in the low BER regions.

Max. Errors: Determines when a measurement is stopped and the algorithm can proceed to the next test point.

Fit Settings:

Trace: Several consecutive measurements can be performed. This is helpful to see the influence of a certain parameter in a setup or simply the pattern dependence. Those measurements are stored in different arrays and displayed as different colored traces in the plot. Each trace can be enabled (check the box "visible") or disabled individually.

Max BER: This is the maximum BER which is taken into consideration for the fitting of the measured data. Measurement points with a BER higher than Max BER are ignored.

Min BER: This is the minimum BER which is taken into consideration for the fitting of the measured data. Measurement points with a BER lower than Min BER are ignored.

Method: Determines whether the inverse error function or a polynomial (which approximates the inverse error function according to reference [2]) is used to extract the jitter contribution (Appendix C). This might be helpful when comparing results with those determined with other instruments.

TJ @: To calculate the total jitter a BER must be given for which the calculation should be carried out.

Plot Settings:

If an analysis has been completed, it automatically scales the plot. The scale can be changed by the user.

Min BER: Used to scale the plot. Sets the minimum displayed BER.

Max BER: Used to scale the plot. Sets the maximum displayed BER.

Min Delay: Used to scale the plot. Sets the minimum displayed time delay value.

Max Delay: Used to scale the plot. Sets the maximum displayed time delay value.

Several global parameters such as trace width and style and default fit settings can be changed in the EA Jitter section of the Setup dialog (Figure 6).





Figure 6 – Jitter Setup

Once everything is set, analysis can be started with the *Start* button. After a short time, a trace should appear on the display similar to that shown in Figure 7.

The results are displayed in the *Result* section of the display, showing random, deterministic and total jitter (peak-to-peak). The fit settings can still be changed to include or exclude certain data points, to change the used method or to calculate the total jitter TJ for a different BER.

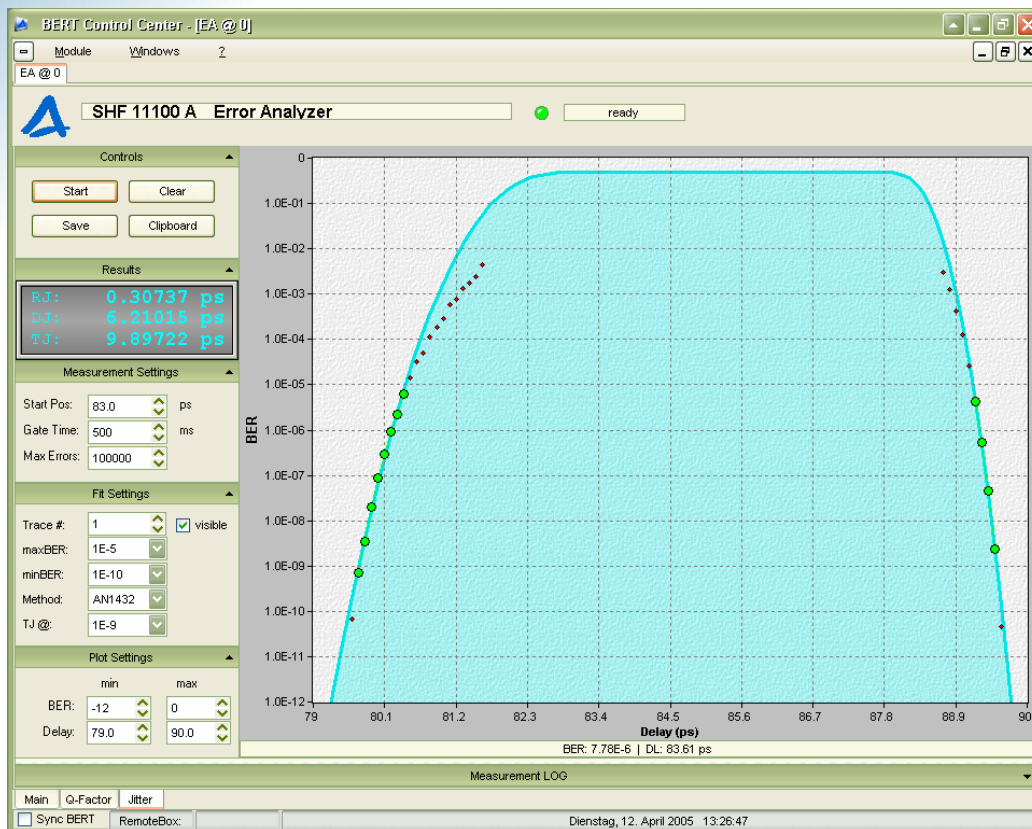


Figure 7 – Jitter Measurement



The random jitter is calculated as the average of both sigma values:

$$RJ = \frac{C_R + C_L}{2} \quad (9)$$

The deterministic jitter is the difference of the mean value of the two distributions:

$$DJ_{pp} = \mu_R - \mu_L \quad (10)$$

Total jitter is calculated using the formula:

$$TJ_{pp}(BER) = \alpha_{BER} \cdot RJ_{RMS} + DJ_{pp} \quad (11)$$

α_{BER} can be calculated using the formula:

$$\alpha_{BER} = 2 \cdot \sqrt{2} \cdot \text{erfc}^{-1}(2 \cdot BER) \quad (12)$$

BER	BER	BER	BER	BER	BER	BER	BER
10 ⁻⁰³	6.18046	10 ⁻⁰⁸	11.224	10 ⁻¹³	14.6976	10 ⁻¹⁸	17.5146
10 ⁻⁰⁴	7.43803	10 ⁻⁰⁹	11.9956	10 ⁻¹⁴	15.3013	10 ⁻¹⁹	18.0265
10 ⁻⁰⁵	8.52978	10 ⁻¹⁰	12.7227	10 ⁻¹⁵	15.8827	10 ⁻²⁰	18.5247
10 ⁻⁰⁶	9.50685	10 ⁻¹¹	13.412	10 ⁻¹⁶	16.4442		
10 ⁻⁰⁷	10.3987	10 ⁻¹²	14.069	10 ⁻¹⁷	16.9876		

The plot can be copied using the *Clipboard* button and pasted into another application; the display can be cleared to start with a new set of measurements; the results and measurement data points can be saved to a file using the *Save* button. For this, plain ASCII or Excel-XML format are available.



$$\begin{aligned}
 BER_R(t) &= \rho_R \cdot \int_{t'=t}^{\infty} PDF_{TJ}(t') dt' & (13) \\
 &= \rho_R \cdot \int_{t'=t}^{\infty} \frac{1}{\sqrt{2 \cdot \pi} \cdot \sigma_R} e^{-\frac{(t'-\mu_R)^2}{2\sigma_R^2}} dt' \\
 &= \frac{\rho_R}{\sqrt{2 \cdot \pi} \cdot \sigma_R} \cdot \int_{t'=t}^{\infty} e^{-\frac{(t'-\mu_R)^2}{2\sigma_R^2}} dt'
 \end{aligned}$$

Now the following variable substitution can be used:

$$x = \frac{t' - \mu_R}{\sqrt{2} \cdot \sigma_R} \quad (14)$$

$$t' = x \cdot \sqrt{2} \cdot \sigma_R + \mu_R \quad (15)$$

$$\begin{aligned}
 \frac{dt'}{dx} &= \sqrt{2} \cdot \sigma_R & (16) \\
 dt' &= \sqrt{2} \cdot \sigma_R \cdot dx
 \end{aligned}$$

Applying the substitution to the variable as well as to the integration limits yields:

$$\begin{aligned}
 BER_R(t) &= \rho_R \cdot \frac{\sqrt{2} \cdot \sigma_R}{\sqrt{2 \cdot \pi} \cdot \sigma_R} \cdot \int_{x=\frac{t-\mu_R}{\sqrt{2} \cdot \sigma_R}}^{\infty} e^{-x^2} dx & (17) \\
 &= \frac{\rho_R}{2} \cdot \frac{2}{\sqrt{\pi}} \cdot \int_{x=\frac{t-\mu_R}{\sqrt{2} \cdot \sigma_R}}^{\infty} e^{-x^2} dx
 \end{aligned}$$

The error function and complementary error function are given by:

$$erf(t) = \frac{2}{\sqrt{\pi}} \cdot \int_{x=0}^t e^{-x^2} dx \quad (18)$$

$$erfc(t) = 1 - erf(t) = \frac{2}{\sqrt{\pi}} \cdot \int_{x=t}^{\infty} e^{-x^2} dx \quad (19)$$

Applying Equation 19 to Equation 17 results in:

$$BER_R(t) = \frac{\rho_R}{2} \cdot erfc\left(\frac{t - \mu_R}{\sqrt{2} \cdot \sigma_R}\right) \quad (20)$$



Appendix B

The approach used separates the random jitter (RJ) and deterministic jitter (DJ) components by fitting the tails of the cumulative distribution functions (Eq. 4 and 5) to the measured BER versus time. The simplifying assumption that the BER of one edge is independent of jitter contribution from the other edge is made.

$$BER_R(t) = N_R \cdot \operatorname{erfc}\left(\frac{t - \mu_R}{\sqrt{2} \cdot \sigma_R}\right) \quad (21)$$

In order to use linear regression as fitting algorithm, Equation 21 is transformed to a linear form:

$$\frac{BER_R(t)}{N_R} = 1 - \operatorname{erf}\left(\frac{t - \mu_R}{\sqrt{2} \cdot \sigma_R}\right) \quad (22)$$

$$1 - \frac{BER_R(t)}{N_R} = \operatorname{erf}\left(\frac{t - \mu_R}{\sqrt{2} \cdot \sigma_R}\right) \quad (23)$$

$$\operatorname{erf}^{-1}\left(1 - \frac{BER_R(t)}{N_R}\right) = \frac{t - \mu_R}{\sqrt{2} \cdot \sigma_R} \quad (24)$$

$$\sqrt{2} \cdot \operatorname{erf}^{-1}\left(1 - \frac{BER_R(t)}{N_R}\right) = \frac{t}{\sigma_R} - \frac{\mu_R}{\sigma_R} \quad (25)$$

Equation 25 has the form $y = m \cdot x + n$ with $m = 1/\sigma_R$ and $n = -\mu_R/\sigma_R$. Now a least squares fitting method is used to minimize the sum of the squares of the differences between the measured data points and the linear fit.

Appendix C

To perform the linear regression on the measurement data, Equation 25 must be used. Some methods use an approximation of the inverse error function by a polynomial to simplify the calculation of Equation 25. To make results comparable with those achieved with other instruments in the *Fit Settings* section of the jitter analysis, *inv. erf* and AN1432 can be selected in the program. If *inv erf* is selected, a built-in inverse error function is used to calculate Equation 25. If AN1432 is selected then the following polynomial approximation is used [2]:

$$\sqrt{2} \cdot \operatorname{erf}^{-1}\left(1 - \frac{BER_R(t)}{N_R}\right) = 1.192 - 0.6681 \cdot \log(BER_R(t)) - 0.0162 \cdot (\log(BER_R(t)))^2 \quad (26)$$



References

- [1] Information Technology – Fibre Channel – Methodologies for Jitter and Signal Quality Specification – MJSQ, International Committee for Information Technology Standardization (INCITS), June 9, 2004
- [2] Agilent Technologies Application Note AN1432 “Jitter Analysis Techniques for High Data Rates”, 2003
- [3] Agilent Technologies White Paper “Jitter Analysis: The Dual-Dirac Model, RJ/DJ, and Q-Scale”, 2005